# INSTABILITY OF THE PERIODIC FLEXURE OF A PANEL SURFACE IN A TURBULENT BOUNDARY LAYER

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The investigation of vibrational instability of a plate in the flow of fluid and gas (panel flutter) is one of the basic problems of the theory of aero- and hydroelasticity [1, 2]. Here we study the instability of the flexure of a flat panel surface, which is in the turbulent boundary layer of an incompressible flow. A simple model of this surface is examined – a series of plates which are infinite in the direction transverse to the flow and which are hinged at the edges. The hydrodynamic part of the problem is solved in the "quasi-laminar" approximation, where the velocity profile of the average flow in the turbulent boundary layer is assigned to the laminar part of the flow. The thickness of the boundary layer is assumed constant along the series of plates. One type of vibrational instability is studied: the periodic flexure of the surface with different signs of the deviation of the adjacent plates. It is known that a region of reduced pressure is formed above a single projection on a plane surface when there is potential flow around it and that the pressure is increased beyond it. Therefore one can assume that variable-sign flexure of the plates will be most typical of the shape of the flexure during steady-state vibrations of the series of plates. That bending shape was chosen in [3] in analyzing supersonic flutter of a periodically fastened large panel (see also the review in [4]).

The problem of the flutter of a series of plates is closely related to the known problem of the stability of a uniform elastic surface with a plane parallel flow around it [5]. The most detailed study of the instability of flexure waves on such a surface was made for laminar flow above it (a Blasius boundary layer, Poiseuille flow, etc.) [5-7]. The problem of determining the response of the average flow in a turbulent boundary layer on the flexure of the surface in the form of a running wave was examined in [8]. However, in [8] an interpolational representation of solutions to Rayleigh's equation was used, which does not allow reliable results to be obtained for the practically important case for "moderate" flexure wave numbers and arbitrary values of the phase velocity of the flexure harmonics. An analogous problem for vibrations of a ship surface in an atmospheric boundary layer was solved on the basis of asymptotic theory [9-12].

Here the asymptotic theory of the stability of shear flows is also used for determining the response of the flow to the periodic flexures of a panel surface. Rayleigh's equation is solved numerically for the profile of velocity perturbations in the boundary layer.

<u>1. Characteristic Equation for Vibrations with Periodic Sign Changes in the Flexure of the Plates.</u> We now examine a series of identical plates (infinite in the direction perpendicular to the flow), each of length L. The edges of the plates are hinged tightly to each other. In the region $y > 0$, a plane parallel flow with a velocity profile $\bar{u}(y)$ flows over the plates in the form of a boundary layer. In the region $y < 0$, the fluid is immobile. The density of the fluid is $\rho_1$ and $\rho_2$ in the regions $y > 0$ and $y < 0$, respectively.

The equations of motion of a thin plate [1] are used to describe the one-dimensional flexure of the surface $y = w(x, t)$

$$\gamma\frac{\partial^2 w}{\partial t^2} + D\frac{\partial^4 w}{\partial x^4} + N\frac{\partial^2 w}{\partial x^2} + 2r\frac{\partial w}{\partial t} = p_2 - p_1, \tag{1.1}$$

where $\gamma$ is the mass per unit area of the plate surface; $D = Eh^3/[12(1 - \mu^2)]$ is the plate bending rigidity (h is the plate thickness, E is Young's modulus, and $\mu$ is Poisson's ratio); r is the phenomenological loss coefficient; N is the external compression; and $p_{1,2}$ is the pressure on the surface from the half spaces $y > 0$ and $y < 0$, respectively.

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The boundary conditions for independent hinging of the plates on the lines $x_n = (n-1)/L \pm 0$ (n is an integer) have the form [1] $w = 0$ and $\partial^2 w/\partial x^2 = 0$. The displacement of the surface of each plate is represented as a Bubnov-Galerkin series:

$$w = \sum_{s=1,2\ldots} w_n^{(s)}(t) \sin[sk_0(x - x_n)] \quad (x_n < x < x_{n+1}) \tag{1.2}$$

where $k_0 = \pi/L$ is the wave number of the main flexure mode, $w_n^{(s)}$ is the amplitude of the s-th mode on the n-th plate. Substituting (1.2) into (1.1) leads to the excitation equations for the modal amplitudes:

$$\frac{\partial^2 w_n^{(s)}}{\partial t^2} + 2r\frac{\partial w_n^{(s)}}{\partial t} + \left(Dk_0^4 s^4 - Nk_0^2 s^2\right) w_n^{(s)} = \frac{2}{L}\int_{x_n}^{x_{n+1}} (p_2 - p_1) \sin[sk_0(x - x_n)]\, dx. \tag{1.3}$$

In order to close the system (1.3) it is necessary to relate $p_{1,2}$ to the surface displacements. In the case of a linear medium, it is sufficient to know the response to elementary flexures $w \sim \exp(ikx - i\omega t)$. If we use the superscript symbol $\wedge$ to denote the complex amplitudes of the variables, we can introduce the input complex elasticity (the hydroelasticity) for the half spaces $y > 0$ and $y < 0$, respectively:

$$Y_{1,2}(c, k) = \pm\hat{p}_{1,2}(c, k)/\hat{w}(c, k) \tag{1.4}$$

where $c = \omega/k$ is the phase velocity of the harmonic $(\omega, k)$. We also can introduce the total complex elasticity of the medium $Y = Y_1 + Y_2$. In this representation, Y, which is real, positive, and independent of the frequency, denotes the presence of a "restoring force" from the side of the fluid. If $Y \sim -\omega^2$, then the action of the fluid reduces to the effect of an attached mass. As opposed to the fluid "conductivity" [6] which is normally used in acoustics, the complex elasticity makes it possible to describe the response for static $(\omega = 0)$ flexures of the surface.

The case of a periodic one-dimensional flexure, where the surface vibrations can be described with the use of a single amplitude A(t): $w_n^{(1)} = (-1)^{n+1}A$, $w(x, t) = A\sin k_0 x$, is the simplest for analysis and at the same time practically important (in view of the discussion above). It is the simplest of the possible forms for periodic flexure, because the flexure for a different relationship of the deflection amplitudes of neighboring plates leads to an infinite complex elasticity. If we assume $A = A_0 e^{-i\omega t}$ it is easy to convince oneself that the system (1.3) reduces to a single equation for $A_0$, from which the characteristic equation is obtained in the form

$$\gamma\omega^2 + 2ir\omega - \left(Dk_0^4 - Nk_0^2\right) = \frac{1}{2} J(c, k_0), \tag{1.5}$$

where $J = Y(c, k) + Y(c, -k)$.

The functions Y(c, k) and J(c, k) are determined for real values of c as an analytic continuation from the integration contour in the inverse Laplace transform, which arises in solving the problem with a given initial flexure of the surface. This makes it possible hereafter to compute the function J for real c and k from the formula $J = Y(c, k) + Y^*(-c, k)$, by analytic continuation of the result in the region of complex c.

An explicit expression for Y can easily be found in the case of one-dimensional potential flow around the surface: $\bar{u}(y) = u_\infty$. A simple calculation gives

$$Y = -\rho_1|k|(c - u_\infty)^2 - \rho_2|k|c^2. \tag{1.6}$$

Correspondingly, Eq. (1.5) takes the form

$$\gamma_0\omega^2 + 2ir\omega - \left(Dk_0^4 - Nk_0^2 - \gamma_1 k_0^2 u_\infty^2\right) = 0, \tag{1.7}$$

where $\gamma_0 = \gamma + \gamma_1 + \gamma_2$, and $\gamma_{1,2} = \rho_{1,2}/k_0$ is the attached mass per unit area from the half spaces $y > 0$ and $y < 0$ for a given type of vibrations.

Hereafter we will consider only the case of stable zero deflection in the absence of flow: $N < Dk_0^2$. By solving (1.7) it is possible to determine the critical flow velocity $u_{\infty c}$, above which static surface instability (divergence [2]) occurs: $u_{\infty c} = \omega_0/k_0\sqrt{\alpha_1}$, where $\alpha_1 = \gamma_1/\gamma_0$ and $\omega_0 = [(Dk_0^4 - Nk_0^2)/\gamma_0]^{1/2}$ is the intrinsic frequency of loss-free vibration of the series of plates for $u_\infty = 0$. For $u_\infty < u_{\infty c}$, the surface vibrations either are neutrally stable in the absence of losses $(r = 0)$ or else attenuate for $r > 0$.

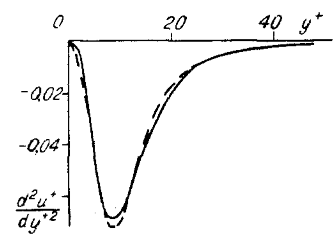


Fig. 1

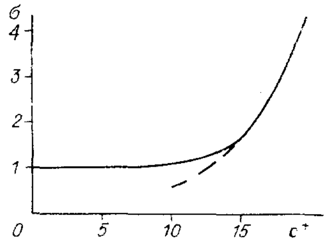


Fig. 2

TABLE 1

| R | $u_*/u_\infty$ | $\delta^+$ |
|---|---|---|
| $4\cdot10^4$ | 0,0381 | 1523 |
| $8\cdot10^4$ | 0,0359 | 2873 |
| $1{,}5\cdot10^5$ | 0,0341 | 5121 |

2. Complex Flow Elasticity in the Turbulent Boundary Layer. In order to investigate the flexure stability of the surface in the turbulent boundary layer, it is necessary to find the complex elasticity of the flow $Y_1(c, k)$. The profile of the average velocity in the turbulent boundary layer has a universal logarithmic section [13]

$$u^+ = \frac{1}{\varkappa}\ln y^+ + B. \tag{2.1}$$

Here $y^+ = yu_*/\nu$; $u^+ = \bar{u}/u_*$, where $u_*$ is the dynamic velocity; $\varkappa = 0.4$; and B is a constant ($B \approx 5.0$ for a hydraulically smooth surface). Equation (2.1) is valid in the region $\delta_L^+ < y^+ < 0.15\delta^+$, where $\delta^+ = u_*\delta/\nu$ and $\delta_L^+ = u_*\delta_L/\nu$ are the dimensionless analogs to the thickness of the boundary $\delta$ and the buffer region next to the wall $\delta_L$, which usually is taken as $\delta_L \approx 30$ [14]. The running distance from the wall $y^+$ is a characteristic scale of the profile (2.1). The function (2.1) vanishes near the wall where the velocity profile tends to become linear ($u^+ \to y^+$) and also in the "wake" region $y^+ > 0.15\delta^+$, where the deviation of $u^+$ from (2.1) can be estimated by adding a small correction term [15]. The appendix gives an analytic approximation for the total velocity profile in the turbulent boundary layer [see (A.4)], which is close to one of the known implicit approximations [16]. Figure 1 shows the profiles of the second derivative computed from Eq. (A.4) (solid line) and for the approximation [16] (dashed line). Use of the explicit representation for the velocity profile (A.4) substantially simplifies the numerical solution of Rayleigh's equation, which contains a singular point. If the Reynolds number $R = u_\infty\delta/\nu$ is given as the initial parameter, it is possible to compute $\delta^+$ and $u_*/u_\infty$ (see Table 1).

In order to apply the asymptotic theory we go to dimensionless quantities and choose $\delta$ and $u_\infty$ as the scales for length and velocity. We denote the dimensionless quantities by their dimensional analogs with a subscript e: $k_e = k\delta$, $\omega_e = \omega\delta/u_\infty$, $c_e = c/u_\infty$, $p_{1,2e} = p_{1,2}/\rho_{1,2}u_\infty^2$, $Y_{1,2e} = Y_{1,2}\delta/\rho_{1,2}u_\infty^2$ etc. The transition from variables with a superscript + to those with a subscript e is determined by the equations

$$y_e = y^+/\delta^+,\quad \bar{u}_e = u^+(u_*/u_\infty),\quad \delta^+ = R\,(u_*/u_\infty). \tag{2.2}$$

Hereafter the index e is omitted for brevity.

We denote the complex profile of the y component of the velocity by $\hat{v}$, which satisfies the Orr–Sommerfeld equation. In the asymptotic theory, $\hat{v}$ is represented as a sum of "inviscous" $\hat{v}_{iV}$ and "viscous" $\hat{v}_V$ solutions [7]. The inviscous solution satisfies the Rayleigh equation

$$\hat{v}_{iV}'' - \left(k^2 + \frac{\bar{u}''}{\bar{u} - c}\right)\hat{v}_{iV} = 0 \tag{2.3}$$

where the primes denote differentiation with respect to y. Strictly speaking, the derivatives of $\hat{v}_{iV}$ deviates substantially from the desired solution in the neighborhood of the layer where $y = y_c$, where $\bar{u}(y_c) = c$. However, the effect of the critical layer can be estimated by making a contour around the singularity in (2.3). As shown in [17], for $k\bar{u}_c' > 0$ [$\bar{u}_c' \equiv \bar{u}'(y_c)$], the contour around the singularity should be made in the lower half plane of the complex y plane.

An analytical continuation of the solution from the contour of integration of the inverse Laplace transform, when the problem is solved with initial conditions (see Sec. 1) leads to same contour around the singularity.

The boundary conditions on the undulating surface can be reduced to the level $y = 0$ by using the standard approach of expanding the solution in a series for small y. These conditions have the form $v = \partial w/\partial t$ and $v' = \bar{u}_0' \partial w/\partial x$, where $\bar{u}_0' \equiv \bar{u}'(0)$. Here it is assumed that the rise of the surface is small compared to the thickness of the buffer region $\delta_L$, which makes up a small part of $\delta$ ($\delta_L \sim 0.01\delta$). Moreover, the rise should be small compared to the viscous scale $\varepsilon = (k\bar{u}_0'R)^{-1/3}$, which in the asymptotic theory is assumed small compared to $\delta_L$. The boundary conditions have been analyzed [7] with a description of the equations of the problem in curvilinear coordinates which are bound to the distorted surface. Here it turns out that the Orr–Sommerfeld equation and the boundary conditions presented above remain in force for a much weaker limitation on the rise of the surface: $kw \ll 1$ (the variables x, y, and $\hat{v}$ take on a different meaning in this case).

The effect of the viscous solution $\hat{v}_V$, which arises from the sticking conditions at the wall, can be considered by specifying an effective boundary condition $\hat{v}_{iV}$ for $y = 0$ (in analogy to [18, 19]). For $k > 0$ and any real c, we obtain

$$\widehat{v}_{iV} + i\omega\widehat{w} = -\varepsilon\widetilde{D}(k, c)\left(\widehat{v}'_{iV} - ik\bar{u}'_0\widehat{w}\right), \tag{2.4}$$

where $z_c = c/\bar{u}_0'\varepsilon$ is the coordinate where the layer coincides with the linear profile $\bar{u} = \bar{u}_0'y$ in units of $\varepsilon$, and $\tilde{D} = -\hat{v}_V(0)/\varepsilon v_V'(0)$ is the generalized Tietjens function. If the velocity profile is linearized after $\hat{v}_V$ is found, then $\tilde{D}(k, c)$ transforms to the Tietjens function in the Holstein definition of $D(z_c)$ [8, 20]. For $z_c > 4-5$, it is possible to use the asymptotic representation, according to which $D \to (1 \pm i)/\sqrt{2|z_c|}$ when $z_c \to \pm\infty$ [20].

Because the phase velocity c is not assumed small hereafter, the results of [7] should be used, in which the problem of constructing the viscous solutions is solved by considering the curvature of the velocity profile. This makes it possible to write the expression for the function $\tilde{D}$ for $c > 0$ in the form

$$\widetilde{D} = \sqrt{\sigma}\, D(\sigma z_c), \quad \sigma = \frac{1}{c}\left[\frac{3}{2}\bar{u}'_0\int_0^{y_c}(c-\bar{u})^{1/2}dy\right]^{2/3}. \tag{2.5}$$

For $c < 0$ it is possible to take $\sigma \approx 1$. The expression for $\sigma$ remains unchanged when it is replaced with variables with the superscript +. Figure 2 shows $\sigma$ as a function of $c^+$, which was constructed for a logarithmic profile (2.1) extrapolated to infinity and corrected near the wall according to (A.4). The dashed line shows the asymptote for sufficiently large $c^+$

$$\sigma = \frac{1}{c}\left[\frac{3}{4}\sqrt{\frac{\pi}{\varkappa}}\, e^{\varkappa(c^+-B)}\right]^{2/3},$$

which is found by substituting the function (2.1) for all $y^+$ into (2.5). Because $\sigma \geqslant 1$, the expression for $\tilde{D}$ for large $z_c$ coincides with the asymptotic $D(z_c)$, but the transition to it occurs at smaller values of $z_c$.

Equation (2.3) should be augmented by the boundary condition [21]

$$\widehat{v}'_{iV} + k\widehat{v}_{iV} = 0\,|_{y=1}. \tag{2.6}$$

In order to calculate $Y_1$ it is also necessary to have an expression for the pressure on the surface $y = 0$ [7]:

$$\widehat{p}_1 = \frac{i}{k}\left[c\widehat{v}'_{iV}(0) + \bar{u}'_0\widehat{v}_{iV}(0)\right]. \tag{2.7}$$

We denote by $\varphi(y)$ the solution to the Rayleigh equation (2.3), which satisfies both the contour around the singularity shown above and the boundary condition (2.6). By substituting $\varphi$ into (2.4) and by using (2.7), we obtain the desired expression for $Y_1$

$$Y_1(c, k) = \left[c\varphi'(0) + \bar{u}'_0\varphi(0)\right]\frac{c - \varepsilon\bar{u}'_0\widetilde{D}}{\varphi(0) + \varepsilon\widetilde{D}\varphi'(0)}. \tag{2.8}$$

We note that $\varepsilon$ in (2.8) is assumed to be rather small, and the viscous function $\tilde{D}(k, c)$ is determined above only for $k > 0$. The transition to complex c in (2.8) is accomplished by analytically continuing $Y_1(c, k)$ as a function of real c.

The fundamental solutions to the Rayleigh equation in the form of a Sommerfeld series in $k^2$ are well known [17]. They can be used to find a relatively simple explicit representation for $Y_1$ for small flexure wave numbers (see [6, 7] for example). In order to determine $Y_1$ in the practically important case of moderate flexure wave numbers ($k \gtrsim 1$) we make use of a numerical solution of the Rayleigh equation, based on the approach in [22, 23]. In order to transform the passage around the contour to a form more convenient for a numerical solution, we represent the desired function $\varphi(y)$ in the form of a linear combination of Tollmien functions $\varphi_a$ and $\varphi_b$ [23]: $\varphi = A_\pm\varphi_a(y) + B_\pm\varphi_b(y)$, where the subscript ± indicates constants for the regions $y > y_c$ and $y < y_c$, respectively. Then the above contour curve takes the form [18]

$$B_+ = B_- \equiv B,\ \ A_+ - A_- = i\pi\left(\bar{u}_c''/\bar{u}_c'\right)B. \tag{2.9}$$

By assuming that the Wronskian of the pair of functions $\varphi_a$ and $\varphi_b$ is strictly equal to −1, we have $A_\pm = \varphi'\varphi_b - \varphi\varphi_b'$, $B_\pm = \varphi\varphi_a' - \varphi'\varphi_a$. By integrating Eq. (2.3) with the condition (2.6) at the initial point $y = 1$ and by using an explicit expansion for $\varphi_a$, and $\varphi_b$ for $y \to y_c$ [23], we express $A_+$ and $B_+$ approximately in terms of $\varphi$ and $\varphi'$ at the point $y = y_c + \Delta$, where $\Delta$ is a small positive quantity. By then determining $A_-$ and $B_-$ with the aid of (2.9), we calculate the values of $\varphi$ and $\varphi'$ which are required to continue the integration at the point $y = y_c - \Delta$.

By choosing $\delta = 0.01 \cdot y_c$ and by using the expansions for $\varphi_a$ and $\varphi_b$ which correspond to the first two and three terms as $y \to y_c$, it is possible to obtain results which are almost independent of $\Delta$. A controlled application of this method to the problem of wind instability gave results coincident with those in [9] (there another procedure was used to integrate around the singularity).

The function $D(z_c)$, which enters Eq. (2.8), has been tabulated in the region $|z_c| < 8$ with the aid of a numerical solution of the equation for $\hat{v}_y$ with a subsequent integration. This gave results which coincide with the standard values [4, 20]. In the range $|z_c| > 8$ an asymptotic formula is used for $z_c \to \pm\infty$.

Figure 3a shows the $\mathrm{Re}\,J_1/k$ and Fig. 3b shows $\mathrm{Im}\,J_1/k$ as functions of c for $R = 8 \cdot 10^4$ (curves 1-4 correspond to k = 0.5, 1.0, 2.0, and 3.0). As $k \to 0$, the function $\mathrm{Re}\,J_1(c, k)$ coincides with the complete function $J_1(c, k)$ in the case of one-dimensional flow: $J_1 = -2k(1 + c^2)$ [see also (1.6)]. The results calculated for $0 < k \lesssim 6$ and the Reynolds number from tables make it possible to suggest a quasipotential model of flow around the vibrating surface, the essence of which is expressed by the approximation

$$\mathrm{Re}\,J_1(c, k) = -2k\,(f + c^2), \tag{2.10}$$

where $f \leqslant 1$ and depends weakly on the Reynolds number. The dependence of f on k is approximated well by the formula $f = \ln(2.05)/\ln(2.05 + k)$. Thus, the drop of the flow velocity within the boundary layer leads to a decrease of the static component of the complex elasticity.

The complex elasticity of the flow in the boundary layer has a small imaginary part, which determines the energy transfer between the flow and the vibrating surface; here the energy enters the surface for $\mathrm{Im}\,Y_1 > 0$ [7]. The presence of regions with $\mathrm{Im}\,J_1 > 0$ in Fig. 3b is caused by the action of a Miles mechanism for strengthening the resonance with the flow of flexure harmonics, whose phase velocities lie in the interval $0 < c < 1$. The harmonics which are not resonant with the flow give a negative contribution to $\mathrm{Im}\,J_1$, which is determined by the excitation of viscous perturbations near the wall ($\tilde{D} \neq 0$). Calculations show that the behavior of $\mathrm{Im}\,Y_1(c, k)$ in the resonance region of the phase velocities almost unaffected by the replacement of $\tilde{D}$ by D in (2.8). Moreover, the behavior $\mathrm{Im}\,Y_1$ is almost the same as for ideal flow ($\tilde{D} \equiv 0$). This agrees with the known results of Miles theory for wind waves [11], because $\mathrm{Im}\,Y_1/k$ coincides with his interaction coefficient of the waves with the wind. According to Miles theory, we also find that this resonance is possible only for sufficiently short surface waves for moderate flexure wave numbers [12].

We note that a correct determination of $Y_1(c, k)$ requires the use of the approximate velocity profile which satisfies the condition $\bar{u}'(1) = 0$. If there is a jump in the first derivative at the boundary with the one-dimensional flow, corrections, which becomes small for $\bar{u}'(1) \ll k|1 - c|$, must be introduced into the boundary condition (2.6). For this reason

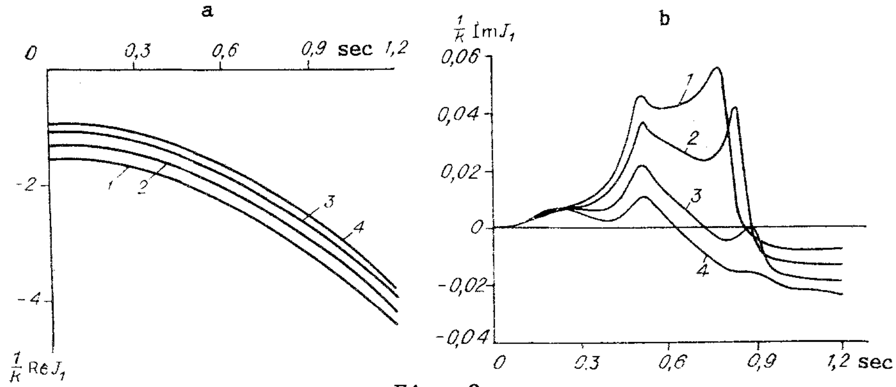


Fig. 3

Re $J_1$ did not converge to the limit of the potential flow for small k in the preliminary calculations for the profile (A.4) with the parameter $\xi = 0$.

3. Resistive Instability of the Periodic Flexure of the Panel Surface. Within the framework of the quasipotential model (2.10), the critical divergence velocity is $1/\sqrt{f}$ times higher than $u_{\infty c}$ obtained in Sec. 1. Because the frequency of the intrinsic surface vibrations becomes zero at the divergence threshold, the phase velocity of the flexure harmonics which accompany the flow drops off in the region with Im $J_1 > 0$ (see Fig. 3b), and the flow velocities are less than the critical velocity. This leads to a subcritical resistive instability of the intrinsic vibrations.

Along with $Y_1$ in the characteristic equation (1.5), there is also a contribution from the complex elasticity of the layer $y < 0$, in which there is no average flow. If we set $\bar{u} \to 0$ and $z_c \gg 1$ in the equations in Sec. 2 and restore the subscript e for the dimensionless quantities, we obtain

$$J_{2e} = Y_{2e}(c_e, k_e) + Y_{2e}^*(-c_e, k_e) = -2k_e c_e^2 \left[1 + k_e \frac{1+i}{(2|\omega_e|R_2)^{1/2}}\right], \tag{3.1}$$

where $R_2 = (\nu/\nu_2)R$, where $\nu_2$ is the kinematic viscosity of the fluid in the half space $y < 0$.

We represent the total functions $J_{1,2e}$ as the sum of 1) their values in the case of potential (quasipotential) flow (around the plates) and 2) the small corrections $\tilde{J}_{1,2}$, which characterize the dissipative processes:

$$J_{1e} = -2k_e(f + c_e^2) + \tilde{J}_1, \quad J_{2e} = -2k_e c_e^2 + \tilde{J}_2. \tag{3.2}$$

In order to solve (1.5) we introduce new variables with a frequency normalization that does not depend on the flow velocity and with a flow velocity normalization in terms of the phase velocity of the flexure waves:

$$\bar{\omega} = \omega/\omega_0, \; V = u_\infty k_0/\omega_0, \; \bar{r} = r/\gamma_0\omega_0. \tag{3.3}$$

Their connection to the dimensionless variables of stability theory (Sec. 2) is determined by the expressions

$$c_e = \bar{\omega}/V, \; \mathrm{R} = V\mathrm{R}_0 \tag{3.4}$$

where $R_0 = \omega_0\delta/k_0\nu$ is the Reynolds number in terms of the phase velocity of the flexure waves. By using (1.5) and (3.2), we obtain a characteristic equation in the new variables:

$$\bar{\omega}^2 + 2i\bar{r}\bar{\omega} - (1 - \alpha_1 f V^2) = \frac{1}{2}(V^2/k_{0e})[\alpha_1\tilde{J}_1(c_e, k_{0e}; \mathrm{R}) + \alpha_2\tilde{J}_2(c_e, k_{0e}; \mathrm{R})] \tag{3.5}$$

where ($\alpha_2 = \gamma_2/\gamma_0$). In the absence of dissipative processes, (3.5) gives vibrations with a frequency $\bar{\omega}^{(0)} = \sqrt{1 - \alpha_1 f V^2}$ which is valid for $V < V_c = 1/\sqrt{\alpha_1 f}$, where $V_c$ is the dimensionless velocity of quasipotential divergence. We now seek the solution of (3.5) for $\bar{\omega}^{(0)} \sim 1$ with the aid of a perturbation method for small $\tilde{J}_{1,2}$ and $\bar{r}$. By calculating the first correction to $\bar{\omega}^{(0)}$ we have the vibration increment

$$\mathrm{Im}\,\bar{\omega} = -\bar{r} + \frac{1}{4}\frac{V^2}{\bar{\omega}^{(0)}k_{0e}}(\alpha_1 \mathrm{Im}\, J_{1e} + \alpha_2 \mathrm{Im}\, J_{2e})_{\bar{\omega}=\bar{\omega}^{(0)}}. \tag{3.6}$$

When a "light" fluid flows over the surface ($\gamma_1 \ll \gamma + \gamma_2$, $\alpha_1 \ll 1$), the divergence threshold is $V_c \gg 1$, and a resistive instability is possible even for $V \ll V_c$. Here the situation is

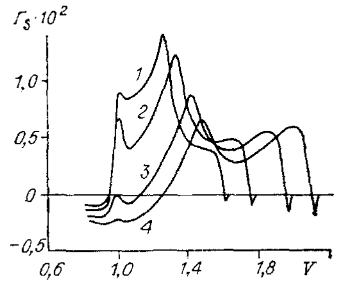


Fig. 4

analogous to the known solution for exciting water waves by wind. Later we will examine the flow of a "heavy" fluid over the surface ($\gamma_1 \gtrsim \gamma + \gamma_2$ and $\alpha_1 \lesssim 1$), where the resistive instability arises for values of V comparable to $V_c$. Moreover, in numerical calculations we limit ourselves to the case of 1) identical fluids on both sides of the surface ($\gamma_2 = \gamma_1$ and $\nu_2 = \nu$) and 2) a relatively small contribution to $\gamma_0$ from the intrinsic mass of the plates ($\gamma/2\gamma_1 \ll 1$), which makes it possible to assume $\alpha_{1,2} \approx 1/2$.

The perturbation method is not valid in this form when $\bar{\omega}^{(0)} \to 0$ ($V \to V_c$). However, by using a linear approximation $\tilde{J}_1 = \beta_0 + i\beta_1 c_e$ for small values of $c_e$ ($\beta_0$ and $\beta_1$ are real coefficients), it can be shown that Eq. (3.6) can be used to estimate the increment even for $V \to V_c$.

By specifying V and R, we can find $\mathrm{Im}\,\bar{\omega}$ for $R_0 = R/V$ by using the graphical data in Fig. 3b. These data, however, are insufficient for constructing a continuous function of $\mathrm{Im}\,\omega$ vs V, because R varies simultaneously with V for a fixed $R_0$. Moreover, the thickness of the boundary layer, which enters into the determination of $k_{0e}$ and R, depends on V. The dependence of $\delta$ on $u_\infty$ and V can be neglected. For example, for the power law $\delta \sim u_\infty^{-1/5}$ [21] a change in $u_\infty$ by a factor of 1.5-2.5, which is characteristic for a region of subcritical instability, leads to a change in $\delta$ by a factor of 1.1-1.2. Calculations show that the dependence of $\mathrm{Im}\,J_{1e}$ on R is weak if $\mathrm{Im}\,J_{1e}$ is positive and sufficiently large. This makes it possible to construct $\mathrm{Im}\,\omega$ as a function of V from the curves in Fig. 3b.

Figure 4 shows the aerodynamic component of the increment $\Gamma_s$ [the term $\sim \mathrm{Im}\,J_{1e}$ in (3.6)], which is constructed for $\alpha_1 = 1/2$ and $R_0 = 8 \cdot 10^4$ in the subcritical region of V values (curves 1-4 correspond to $k_{0e}$ = 0.5, 1.0, 2.0, and 3.0). The calculation includes the dependence of $\mathrm{Im}\,J_{1e}$ on R.

Estimates from the curves in Fig. 3b for the magnitude of $\mathrm{Im}\,J_{2e}$, which determine the losses in the immobile fluid, give a value of $\sim 2 \cdot 10^{-3}$, which allows this contributuion in (3.6) to be neglected. The functions shown in Fig. 4 make it possible to estimate the level of losses in the plate material, which are necessary for suppressing the instability. By relating $\bar{r}$ to the Q for the vibrations of a free surface (in a vacuum) with this form of the flexure, $Q_f = \mathrm{Re}\,\omega_f/2\ \mathrm{Im}\ \omega_f$, where $\omega_f$ is the frequency of free vibrations, it is possible to write the condition for suppressing the instability in the form

$$\Gamma_s < \bar{r} = \frac{1}{2Q_f}\sqrt{\frac{\gamma}{\gamma_0}}. \tag{3.7}$$

For example, if $\Gamma_s \sim 10^{-2}$ and $\gamma/\gamma_0 \sim 0.2$, we obtain from (3.7) that the instability vanishes for $Q_f < 23$.

Thus, the increment in the instability of a periodic sign-varying flexure in a series of plates can be expressed in terms of the complex elasticity of the flow for two harmonics of this flexure. For moderate flexure wave numbers, a quasipotential flow is realized over the surface, which leads to an increase in the critical divergence velocity compared to the case of one-dimensional flow. The resonance interaction of the surface vibrations with the flow in the critical layer (the Miles mechanism) determines the occurrence of a resistive instability of the surface at velocities lower than the critical velocity. Here we have found the condition for suppressing the resistive instability due to the losses in the plate material. We add that determining the boundaries for applying the single-mode model is approximately related to the analysis of the double-mode for periodic flexure with a period 2L. It can be shown that, for a homogeneous flow, the effect of the second mode is small for $u_\infty < 1.5u_{\infty c}$. Here the resonance mechanism for building up the second flexure mode of the plate is also insignificant, in view of the large phase velocity of the surface flexure harmonics which are produced by it.

Appendix. Several approximations are known for the velocity profile in a turbulent boundary layer [13-16]. However, almost all of them start from an explicit representation for the first derivative of the profile at the wall. The same velocity profile should be determined numerically. The explicit Reichardt approximation ([13], p. 547) does not satisfy the condition that the third derivative must be zero at the wall; it gives a deviation of ~25% from the linear law at the boundary of the viscous sublayer $y^+ = 5$. The implicit approximation [16]

$$\frac{du^+}{dy^+} = \left\{1 + \varkappa y^+\left[1 - \exp\left(-\frac{8}{\varkappa}10^{-4}y^{+2}\right)\right]\right\}^{-1} \tag{A.1}$$

gives an analogous deviation of ~8%, which to a larger degree corresponds to experimental data. Here we present an explicit approximation of the velocity profile, which is close to (A.1).

For $y^+ \ll \delta^+$, we write the following expression for the derivative of $u^+$:

$$\frac{du^+}{dy^+} = \frac{1}{1+\varkappa y^+} + \varkappa y^+ e^{-\lambda y^+} + b y^{+2} e^{-\lambda_1 y^+} \tag{A.2}$$

where b, $\lambda$, and $\lambda_1$ are parameters. For $y^+ \to \infty$, (A.2) approaches Eq. (2.1), and $B = (\varkappa/\lambda^2) + (2b/\lambda_1^3) + (1/\varkappa)\ln\varkappa$. It follows from (A.2) that $du^+/dy^+ = 1$ and $d^2u^+/dy^{+2} = 0$ at $y^+ = 0$. Then by applying the condition that $d^3u^+/dy^{+3} = 0$ at $y^+ = 0$, we can express $\lambda_1$ and b in terms of $\lambda$ and B:

$$b = \varkappa(\lambda - \varkappa),\ \lambda_1 = [2b/(B - \varkappa/\lambda^2 - \ln\varkappa/\varkappa)]^{1/3}. \tag{A.3}$$

The results of constructing the second derivative of the velocity profile for $\lambda = 0.2285$ and $B = 5.0$ are shown in Fig. 1.

The additive correction term to (2.1) in the wake region is written in Coles' form [14, 15]. As a result, we obtain an approximation for the total velocity profile ($0 < y^+ < \delta^+$):

$$u^+ = \frac{1}{\varkappa}\ln(1+\varkappa y^+) + B - \frac{1}{\varkappa}\ln\varkappa - q(y^+) + h_m \sin^2\left[\frac{\pi}{2}(1+\xi)\frac{y^+}{\delta^+}\right].$$

Here $q = \varkappa e^{-\lambda y^+}\left(\frac{y^+}{\lambda} + \frac{1}{\lambda^2}\right) + b e^{-\lambda_1 y^+}\left(\frac{y^{+2}}{\lambda_1} + \frac{2y^+}{\lambda_1^2} + \frac{2}{\lambda_1^3}\right)$. For a boundary layer with no pressure gradient, $h_m = 3.1$ [15]. As oppposed to [15], the correction term in (A.4) is shown in a shifted form. The introduction of the parameter $\xi$ makes it possible to fulfill the condition of a zero first derivative of $u^+$ at the boundary with the one-dimensional flow $y^+ = \delta^+$ (see Sec. 2). If $h_m = 3.1$, this condition gives $\xi = 0.1476$.

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# MODELING SEPARATION BOUNDARIES IN THE FLOW OF A HEAVY FLUID OVER A WING PROFILE

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The presence of boundaries separating media has a substantial effect on the nature of the flow around a wing and the forces acting on it. In the plane case the separation boundary is a flow curve, where velocity undergoes a tangential jump when it passes through it, so a piecewise analytic function with an unknown curve of discontinuity in the region of the flow must be sought in order to construct a complex flow potential. If the [wing] contour is sufficiently far from the separation boundaries, modeling the separation lines by continuously distributed singularities within the framework of the theory of low-amplitude waves makes it possible to solve a wide circle of problems [1-4], in which the conditions on the contour are fulfilled exactly.

Here the method of distribution of singularities is used to solve problem in which a two-layered heavy fluid with a free surface flows around a profile located in the layer of the fluid. This problem is related to the "dead wave" phenomena [5], which is caused by the formation of waves on the boundary separating fluids of different density. We make note of [6], where an attempt was made to solve the problem by the method in [7]. However, the investigation in [6] was limited by the choice of an integral equation; only a contour of continuous curvature was studied; and the problem of defining the circulation was not treated.

1. In a system of coordinates bound to the [wing] profile C, we examine a steady-state flow of an ideal incompressible heavy fluid, which is limited by a free surface $E_1$ and consists of a layer of thickness H of density $\rho_1$ and an infinitely thick layer of density $\rho_2$ with a boundary $E_2$ separating the fluids. The Ox axis is directed against the flow; the Oy axis is vertical upwards, and the origin of the coordinates lies in the middle of the chord of C. The flow velocity at infinity ahead of the profile is parallel to the immobile separation boundary and is equal to $-V_j$ ($j = 1, 2$).

In the representation of a potential flow, the problem reduces to determining the complex potentials of the perturbed flow $W_j^0(z)$ in the corresponding regions $D_j$, where $D_1$ represents

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